6. LINEARIZATION TECHNIQUES

6.1. Feedforward
6.2. Feedback
6.3. Predistortion
6.4. Multi-tanh
6.5 Derivative superposition
6.6. Image rejecting filters for killing harmonics

ACCURACY REQUIREMENTS OF CANCELLING SYSTEMS

One method of reducing distortion is to try to cancel it with distortion equal in amplitude but opposite in phase. However, the accuracy requirements for this are quite tough. Using cosine rule for signal vector, the achievable cancellation is

\[CANC = 10 \cdot \log \left(1 - 2(1 + dA/A)\cos(\Delta \phi) + (1 + dA/A)^2\right)\]

where \(\delta A\) and \(\Delta \phi\) are amplitude and phase errors, respectively. For example, to achieve 30 dB reduction (1/30 in amplitude) in distortion, amplitude error must be less than 0.25 dB (3%) and phase error less than 1 degree. These requirements are similar but tougher than for SSB upconverters.
EXAMPLE: IQ QUANTIZATION IN PREDISTORTERS

Suppose that we have a polynomial predistorter where IM3 distortion can be programmed with I and Q coefficients. Now the output is quantized which means that they may differ from the desired value by 0.5 lsb. This appears as residual, uncancelled distortion.

The achievable cancellation of distortion term in this case is related to programming word length by

\[
CANC = 20 \cdot \log\left(\left|\frac{\text{coeff}}{1 - N^{-1/2}}\right|\right) \quad -1 < \text{coeff} < 1
\]

Thus, quantization limits the cancellation accuracy of small IM3 vectors.

FEEDFORWARD

- Invented by Black (Bell Labs) 1928
- The distortion generated by the main amplifier A1 is extracted, amplified by an auxiliary error amplifier A2, and subtracted from the output signal
- To achieve good cancellation in node B, the error amplifier A2 needs to be very wideband and memoryless. This calls for wideband input and output matching
- Adaptation is tricky.
FEEDBACK

- Basic principle by Black in 20’s
- Distortion is in the forward branch reduced by 1/T, where T is the loop gain of the amplifier feedback combination. Distortion in the feedback branch appears directly in the output.
- Fundamental problem of using output that already exists to correct the input that caused that distortion -> works well only with periodic signals
- Stability and bandwidth issues
- TIM (Transient intermodulation distortion, Otala in 70’s) with signals that are fast compared to loop bandwidth

![Feedback Diagram](image)

CARTESEIAN FEEDBACK

At RF frequencies it is difficult to achieve very high loop gain. One way to circumvent this is to form the error signal at baseband and use quadrature up and down conversion in the direct and feedback branches.

Problems:
- delay in mixers and PA reduce the achievable bandwidth
- noise and linearity of the feedback mixers affect directly the output

Papers: Faulkner.
BOOSTED FEEDBACK

To improve the linearity of audio power amplifiers, H. Sjölund has proposed a boosted feedback technique ("Feedback Boosting Negative Impedance"), where the loop gain of the amplifier is increased to infinity using negative resistors. Same care is needed to maintain stability, but low-frequency linearity can be increased noticeably.

Example:
The transfer func. of the inverting opamp amplifier is

\[
\frac{v_o}{R_2} = -\frac{v_{in}}{A_{vo}} \left( \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} \right)
\]

The dependency to opamp gain \( A_{vo} \) disappears, if

\[ R_3 = -(R_1 || R_2) \]

and in the same time, the distortion produced by the opamp is reduced to zero. The negative impedance can be implemented with the circuit below (H. Sjöland, Lund Univ. 1997)

PREDISTORTION (MEMORYLESS)

The basic idea of predistortion is to cancel the distortion in the power amplifier by predistorting the transmitted signal with the inverse function of the amplifier. I.e. if the amplifier is driven to compression, higher amplitudes need to be expanded to make the total response linear.

This compensating nonlinearity makes the spectrum of the transmitted signal wider, requiring higher sampling rates, wider IF filters etc.
EXAMPLE: A POLYNOMIAL 5TH ORDER PREDISTORTER

5th order distortion is created by multiplying the input signal by a programmable 4th order polynomial. In baseband predistorter, complex multiplication is needed to correct AM-PM. Also IQ imbalance and phase errors of the IQ modulator need to be corrected before upconversion (Faulkner: Crisis-network).

In IF predistorter the complex multiplication can be performed by tuning I and Q components simultaneously before they are summed up.

PREDISTORTION WITH MEMORY

Memory in power amplifiers appears as hysteresis in AM-AM and AM-PM curves, making memoryless predistortion less effective. To compensate this, some memory in the envelope following capacity is needed.

Another problem with simple predistorters is that they create equal IM3 sidebands only. If the power amplifier has unequal sidebands, only one of them can be compensated at a time.
MULTI-TANH PRINCIPLE

Multi-tanh principle (introduced by B. Gilbert, IEEE j. Solid-State Circuits, ) is a new name for old idea to use several nonlinear transfer characteristics that are slightly offset. Especially this has been employed in BJT differential pairs, where connecting several differential pairs in parallel and offsetting them some tens of mV either by dimensioning or voltage generators, a highly linear input range can be generated, while the gain can still be continuously tuned by varying the bias current. These have been used in many RF products by Analog Devices.

Next, a conventional unlinearized differential pair is compared to 2-stage multi-tanh pair. The idea can be extended to more parallel stages.
DERIVATIVE SUPERPOSITION

FET type transistors exhibit sign reversal of K3 at some bias point. Haigh et al. have used this to reduce distortion: they have several transistors in parallel, scaled and operating at different gate bias voltages so that some transistors have positive and some negative K3. This method has been formulated as “derivative superposition”: desired shape for K3 is synthesized, and by integrating it, K2, K1, and I-U curves can be generated.

This technique is especially suited for distributed amplifiers where naturally several transistors are used. The same idea can be used to create frequency multipliers: fundamental is suppressed by substracting the outputs of M1 and M2, while the desired harmonic is amplified.
ASYMMETRIC POLYPHASE FILTERS

Polyphase RC filters, when driven by quadrature signals, have asymmetric frequency response. This can be exploited in mixers for attenuating image band, or in nonlinear oscillators for killing harmonics. Suppose two quadrature squarewave signals \( x_I \) and \( x_Q \)

\[
x_I(t) = \frac{4}{\pi} \left( \cos(\omega t) - \frac{1}{3} \cos(3\omega t) + \frac{1}{5} \cos(5\omega t) - \cdots \right)
\]

\[
x_Q(t) = \frac{4}{\pi} \left( \sin(\omega t) + \frac{1}{3} \sin(3\omega t) + \frac{1}{5} \sin(5\omega t) - \cdots \right)
\]

Using Euler equations for \( \sin \) and \( \cos \), \( x_I + jx_Q \) results in asymmetric (complex) spectrum

\[
x(t) = \frac{4}{\pi} \left( e^{j\omega t} \frac{1}{3} e^{-j3\omega t} + \frac{1}{5} e^{j5\omega t} - \frac{1}{7} e^{-j7\omega t} + \cdots \right)
\]

Thus, 3rd, 7th, 11th, and in general, \((4k-1)\)th harmonic is at negative frequencies and can be filtered by asymmetric filters that have stopband only at negative frequencies.
7. NOISE IN NONLINEAR CIRCUITS

Some basic considerations

- Noise sources are bias and thus signal dependent (so-called cyclostationary noise).
- Noise gain is signal dependent.
- Usually, noise is small enough not to affect the large-signal behaviour.
- Nonlinearity means time-domain multiplication, i.e. frequency domain convolution. Convolution of discrete spectrums is easy, but convolution of wideband spectrums is heavy.
- Phase noise deals with phase, which is not directly any circuit voltage or current.

Some classes of circuit

- Linear amplifiers: linear noise analysis
- Mixers: frequency conversion analysis
- Oscillators: algorithm for showing phase as node voltage
- Nasty ones: companding log-domain filters, analog predistorters, RF power amplifiers. New algorithms are being developed.
- Circuits where noise affects circuit behaviour: transient noise analysis

LINEAR NOISE ANALYSIS (SMALL-SIGNAL AMPLIFIERS)

- All noise sources are multiplied by the transfer function from the source to the output and summed as noise powers.
- Fast.
- Does not handle any nonlinearities
- Usually does not handle correlated noise sources
- Noise aliasing in sampling can be handled quite easily

\[
\begin{align*}
    u_{on} &= \sqrt{((R2/R1)u_1)^2 + ((1+R2/R1)u_2)^2 + u_3^2)} \\
    u_{in} &= u_{on} / Au(f)
\end{align*}
\]
NOISE SOURCES IN TRANSIENT ANALYSIS

- Random and even signal-dependent noise sources are easy to build in transient analysis
- Suffers from high numerical noise. Thus, deterministic tones are often better than white noise.
- Beware macromodels. Terminal currents contain displacement currents ($i = C \frac{dv}{dt}$) that do not affect noise. Signal-dependent noise generators can also cause convergence problems.

- Summing noise as signals into transient analysis can be considered as a Monte-Carlo test, many of which are needed to get good estimate for noise spectrum.

FREQUENCY CONVERSION ANALYSIS

- Applied for periodically forced circuits (mixers, large-signal amplifiers)
- Gain can be considered as a periodic function that is described as a discrete line spectrum. All tones in the line spectrum mix signal and noise down to baseband.
- Signal-dependent noise sources can be lumped in time-varying gain.
Time-varying conductances and capacitances are expanded into Fourier coefficients. Noise voltages at different sidebands can be obtained by convolving the noise current spectrum $i$ with the line spectrum of the nonlinear components:

\begin{align*}
    I_{1l} &= \begin{bmatrix}
        \delta_{m0} & \delta_{m1} & \delta_{m2} \\
        \delta_{m1} & \delta_{m0} & \delta_{m1} \\
        \delta_{m2} & \delta_{m1} & \delta_{m0}
    \end{bmatrix}
    \begin{bmatrix}
        V_{1l} \\
        V_{b} \\
        V_{1u}
    \end{bmatrix} \\
    I_b &= \begin{bmatrix}
        0 & 0 & 0 \\
        \omega_0 & 0 & 0 \\
        \omega_0 + \omega & 0 & 0
    \end{bmatrix}
    \begin{bmatrix}
        \delta_{m0} & \delta_{m1} & \delta_{m2} \\
        \delta_{m1} & \delta_{m0} & \delta_{m1} \\
        \delta_{m2} & \delta_{m1} & \delta_{m0}
    \end{bmatrix}
    \begin{bmatrix}
        V_{1l} \\
        V_{b} \\
        V_{1u}
    \end{bmatrix} \\
    I_{1u} &= \begin{bmatrix}
        0 & \omega_0 & 0 \\
        \omega & 0 & \omega_0 + \omega \\
        0 & \omega_0 + \omega & 0
    \end{bmatrix}
    \begin{bmatrix}
        \delta_{m0} & \delta_{m1} & \delta_{m2} \\
        \delta_{m1} & \delta_{m0} & \delta_{m1} \\
        \delta_{m2} & \delta_{m1} & \delta_{m0}
    \end{bmatrix}
    \begin{bmatrix}
        V_{1l} \\
        V_{b} \\
        V_{1u}
    \end{bmatrix}.
\end{align*}

This results in a $M \times N$ matrix, where $M$ is the number of nodes and $N$ the number of desired sidebands.
PHASE NOISE IN AUTONOMOUS OSCILLATORS

- Due to amplitude limiting, amplitude noise decays in oscillators to the stable trajectory, but
- There is no restoring mechanism for phase errors. Thus, phase error caused by noise cumulates indefinitely. Because of this
- Phase noise has infinite memory, that needs to be modelled separately by an integrating response:

$$S_\phi(\Delta f) = \frac{K}{\Delta f}$$

where $\Delta f$ is the offset from the center frequency.

Especially in nondsinusoidal oscillator (e.g. ring oscillators) the phase shift caused by noise impulse is a clear function of time.

HAJIMIRI'S PHASE NOISE ANALYSIS

Consider noise as impulses that are occurring at random time moments. If noise appears at time A and the oscillator is not narrowband (e.g. ring oscillator), the noise decays before next transition. Instead, when noise impulse appears during the transition (B) it appears as permanent phase shift.

Measuring the gain from noise impulses into phase error, so-called impulse sensitivity function ISF can be derived (it closely resembles the derivative of the waveform). This can be expanded into Fourier series, the coefficients $c_i$ of which determine the gain from a given noise sideband into the vicinity of carrier.

$$ISF(\omega_0 \tau) = \frac{c_0}{2} + \sum_{n=1}^{\infty} c_n \cos(n\omega_0 \tau + \theta_n)$$

$$\delta(t) = \frac{1}{q_{max}} \left( \frac{c_0}{2} \int_{-\infty}^{t} i(\tau) d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau) \cos(n\omega_0 \tau) d\tau \right)$$
Often, upconverted 1/f noise increases phase noise. The equation suggests that 1/f is mainly upconverted by the dc term of $c_0$. Thus it is advantageous to minimize $c_0$, which is achieved by having maximally symmetrical waveform. Thus, e.g. in ring oscillators, unequal rise and fall times may considerably increase phase noise by upconverting 1/f noise.

\[
\phi(t) = \frac{1}{q_{max}} \left( \frac{c_0}{2} \int_{-\infty}^{t} i(\tau)d\tau + \sum_{n=1}^{\infty} c_n \int_{-\infty}^{t} i(\tau)\cos(n\omega_0 \tau)d\tau \right)
\]

HAJIMIRI CONT ...

NON MONTE-CARLO TIME DOMAIN
NOISE ANALYSIS

Demir and Sangiovanni-Vincentelli have introduced a method, where time evolution of noise power is calculated in parallel with signal evolution, by solving stochastic differential equations (they need different solvers due to different handling of differentials $dv$, $di$). The result is directly the time evolution of noise power, not just the response of a single noise sequence.

The method is essentially a transient analysis with quite heavy computational load. However, it is quite general and applicable to phase noise analysis, too.

Further Reading:

- S. Szczepanski, R. Schauman: Effects of Weak Nonlinearities in Tranconductance-Capacitor Filters. 4pp.