Abstract—In this paper we analyze capacity losses in advanced wideband Code Division Multiple Access (WCDMA) networks due to imperfections in the estimation of the system and channel parameters. Key components in the analysis are multiple access intracell interference (MAI), intercell interference, efficiency of the interference cancellation, multipath intensity profile, efficiency of the rake combiner, inefficiency of signal and channel parameter estimation and additive white Gaussian noise. Receiver structures are based on a conventional matched filter rake combiner that can be supported by either linear or nonlinear interference canceler. Both the equal gain and maximal ratio combining techniques are considered. Results show that significant capacity gains can be achieved by interference cancellation in comparison to the conventional techniques. However, the capacity gain may be completely lost in fast fading channels due to the estimation imperfections.

I. INTRODUCTION

Capacity of CDMA systems has been extensively simulated and analyzed through recent years, e.g., [1]. Multiuser detection [2] has also been a challenging research area for longer than a decade. However, much less literature is available on the combined effect of these areas, e.g., [3-5]. One reason for such a situation has been a lack of a systematic mathematical framework for this kind of evaluation. In [6] we have proposed one candidate approach. The analysis is complicated because there are many parameters involved and some of the system components are rather complex resulting into an imperfect operation. The analysis of an advanced wideband CDMA network should in general take all key system elements into account including their imperfections. Due to the complexity of this task for large number of active users some approximations and simplifications are usually necessary. In this paper we present some numerical results and comparisons relying the framework model of [6] extended to a conventional rake receiver and a linear decorrelator. First, capacity formulation for a matched filter (or correlator) rake receiver is derived. The same procedure is repeated for more advanced systems employing either linear decorrelator or iterative nonlinear intracell interference cancellation schemes. Then the average rake receiver efficiency is defined for equal gain and maximal ratio combiners.

Sensitivity of Advanced Wideband CDMA Network Capacity to Various Channel and System Parameter Imperfections

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Fig. 1. System receiver block diagram.

Sensitivity function is defined in order to quantify the capacity loss. In numerical results section capacity and sensitivity curves are presented in two chosen multipath channels for all three studied receiver structures.

II. SYSTEM MODEL

The received signal is directed to a bank of matched filters matched to the spreading codes of different users. In the case of conventional receiver the output giving the highest peak is fed to the rake combiner exploiting the multipath diversity. For channel estimation (phase, code delay and amplitude) either continuous pilot channel or time-multiplexed symbols can be used in the rake combiner. When interference cancellation is included it can be performed either before or after the rake combining (pre- or postcombining). Finally, the data is demodulated, deinterleaved and decoded until the detected bits are available. A generic block diagram of the receiver structure is depicted in Fig. 1. This paper concentrates on the thick line wrapped blocks.

A. Channel Model

The exponential multipath intensity profile (MIP) is a widely used analytical model realized as a tapped delay line [7]. It is very flexible in modeling different propagation scenarios. The decay of the profile and the number of taps in the model can vary. Averaged power coefficients in the multipath intensity profile are

\[ \alpha_l = \alpha_0 e^{-\lambda l}, \quad l, \lambda \geq 0 \]  

where \( \lambda \) is the decay parameter of the profile and \( l \) is the multipath index. Power coefficients in the model are normalized as

\[ \sum_{l=0}^{L-1} \alpha_l e^{-\lambda l} = 1. \]
When $\lambda = 0$ the profile is flat and it becomes more impulse-like as $\lambda$ increases. The number of resolvable paths $L$ depends on the channel sampling rate and delay spread. Table I depicts channel profiles used in this paper.

### III. Capacity Analysis

The starting point in the evaluation of CDMA system capacity is $y_m = E_{bm}/N_0$, the received signal energy per symbol over overall noise density in a given reference receiver with the index $m$. For the purpose of this analysis we can represent this parameter in general case as

$$y_m = \frac{E_{bm}}{N_0} = \frac{ST}{(1 + \nu)I_{oc} + I_{oin} + \eta_b} = \frac{ST}{(1 + \nu)I_{oc} + \sigma_n^2} \tag{3}$$

where $I_{oc}$ and $I_{oin}$ are power densities of intercell and overlay type internetwork interference, respectively, and $\eta_b$ is thermal noise power density. $S$ is the overall received power of the useful signal and $T = 1/R_b$ is the information bit interval. Intercell interference is modeled by a fraction $\nu$ of the intracell interference. Contributions of $I_{oin}$ and $\eta_b$ to $N_0$ are combined by introducing a Gaussian noise variance term $\sigma_n^2$.

#### A. Conventional Rake Receiver

For a conventional single-user rake receiver (3) can be rewritten in form

$$y_m = \frac{r_m^{(L_0)}}{(1 + \nu)f(\alpha)p^2K_{MF} + \sigma_n^2} \tag{4}$$

where $r_m^{(L_0)}$ refers to the rake efficiency of the desired user (i.e., received energy by the rake, index $L_0$ is the number of rake fingers), $K_{MF}$ is the average number of active users that can be served at the predefined SNR target $Y_0$, $p^2$ is the expectation of squared cross correlations (often approximated as inversely proportional to the processing gain $G$), $f(\alpha) = E[I_{oc}] = w_{nr}I_0L\alpha$ is the intracell interference (MAI) density, $w_{nr}$ is the rake combiner coefficient and $\alpha = 1/L$ is the average multipath power.

Maximum number of simultaneous active users (= capacity at $y_m = Y_0$) can be solved from (4) to be

$$K_{MF} = \frac{r_m^{(L_0)} - Y_0\sigma_n^2}{Y_0(1 + \nu)f(\alpha)p^2} \tag{5}$$

#### B. Decorrelating Rake Receiver

When a simple linear multiuser receiver is used the intracell interference can be completely eliminated at the expense of enhanced overall noise variance. Mathematically the required signal-to-noise ratio can be formulated as

$$y_m = \frac{r_m^{(L_0)}}{(1 + \nu)f(\alpha)\rho^2K_{DEC} + \sigma_n^2\rho^2\sigma_n^2} \tag{6}$$

where $K_{DEC}$ is the number of active users that can be supported when the linear decorrelating rake receiver is employed. $\sigma_n^2$ is the additional variance due to the estimation errors derived in the Appendix and $\rho^2\sigma_n^2$ is the $n$th diagonal element of the inverted correlation matrix (noise enhancement). Directly solving from (6), at $y_m = Y_0$, $K_{DEC}$ becomes

$$K_{DEC} = \frac{r_m^{(L_0)} - \sigma_n^2\rho^2\sigma_n^{-1}}{(1 + \nu)f(\alpha)p^2\rho^2\sigma_n^{-1}} \tag{7}$$

Analytical evaluation of $\sigma_n^{-1}$ will be impractical for large user populations. As a simplification we look at the average performance with fixed positive correlation coefficients in $\rho$, i.e., $R_{ij} = \rho, \forall i \neq j$ and $R_{ii} = 1$. With this approximation the noise enhancement $\sigma_n^{-1}$ follows values gathered in Table II.

#### C. Rake Receiver with Nonlinear Interference Cancellation

Sometimes nonlinear (multistage [2, 8]) multiuser detection is more feasible than linear interference cancellation (e.g., long scrambling codes used in the spreading). Channel and interference estimation is included in the process. At least the strongest interference signals are supposed to be estimated and cancelled. Signal-to-noise ratio in this case can be represented as

$$y_m = \frac{r_m^{(L_0)}}{(1 + \nu)f(m, \alpha, r)p^2K_{NL} + \sigma_n^2} \tag{8}$$

### Table I

<table>
<thead>
<tr>
<th>Path</th>
<th>$\lambda = 0.5$</th>
<th>$\lambda = 1$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0 dB</td>
<td>0 dB</td>
<td>0 dB</td>
</tr>
<tr>
<td>1 dB</td>
<td>-2.17 dB</td>
<td>-4.34 dB</td>
</tr>
<tr>
<td>2 dB</td>
<td>-4.34 dB</td>
<td>-8.69 dB</td>
</tr>
<tr>
<td>3 dB</td>
<td>-6.51 dB</td>
<td>-13.03 dB</td>
</tr>
</tbody>
</table>

### Table II

<table>
<thead>
<tr>
<th>Correlation Coefficient</th>
<th>Noise Enhancement</th>
<th>Correlation Coefficient</th>
<th>Noise Enhancement</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\rho = 0$</td>
<td>$\sigma_0$</td>
<td>$\rho = 0.1$</td>
<td>$\sigma_0$</td>
</tr>
<tr>
<td>$\rho = 0.15$</td>
<td>$\sigma_0$</td>
<td>$\rho = 0.2$</td>
<td>$\sigma_0$</td>
</tr>
<tr>
<td>$\rho = 0.2$</td>
<td>$\sigma_0$</td>
<td>$\rho = 0.3$</td>
<td>$\sigma_0$</td>
</tr>
</tbody>
</table>
where the intracell interference density

\[
f(m, c, r) = \frac{1}{K} \sum_{k=1}^{K} \sum_{r=1}^{L} \sum_{l=1}^{L} \sum_{k=m}^{L} w_{mr} \alpha_{kl} (1 - C_{kl}) + \frac{1}{K} \sum_{r=1}^{L} \sum_{l=1}^{L} \sum_{m}^{L} w_{mr} \alpha_{ml} (1 - C_{ml})
\]  

(9)

Interference cancellation efficiency in (9) can be represented in the form [6]

\[
C_{kl} = 2(1 + \varepsilon_a)(1 - 2\varepsilon_m)(1 - \sigma_0^2) - (1 + \varepsilon_a)^2
\]  

(10)

where \(\varepsilon_a\) consists of amplitude and code delay estimation errors, \(\varepsilon_m\) is the bit error probability and \(\sigma_0^2\) is the carrier phase tracking error variance. When PSK modulation is used in the AWGN channel \(\varepsilon_m = 1/2erfc(\sqrt{y_m})\) and equations (8)-(10) can be solved through iterative process starting from the initial value \(C_{kl} = 0, \forall m, k, l\).

D. Rake Receiver Efficiency

The parameter \(r_m(L_0)\) in (4)-(8) called rake receiver efficiency is defined in [6]. It corresponds the bit energy captured by the rake combiner. Average rake efficiency for the equal gain combiner (EGC) is

\[
E_{\{r_m(L_0)\}} = E \left[ \left( \sum_{r=1}^{L_0} \cos \varphi_r \sqrt{\alpha_r} \right)^2 \right]
\]

\[
= E \left[ \left( \sum_{r=1}^{L_0} (1 - \varepsilon_{cr}^2/2) \sqrt{\alpha_r} \right)^2 \right]
\]

\[
= \sum_r \sum_{l=1}^{L} (1 - \sigma_{0r}^2)(1 - \sigma_{0l}^2) \alpha_r \alpha_l + \sum_r (1 - 2\sigma_{0r}^2 + 3\sigma_{0r}^2) \alpha_r
\]  

(11)

For the maximal ratio combiner (MRC) taking the upper limit [6] the same relation becomes

\[
E_{\{r_m(L_0)\}} = E \left[ \left( \sum_{r=1}^{L_0} \sqrt{\alpha_r} \right)^2 \right]
\]

\[
= \sum_{r=1}^{L_0} \frac{\alpha_r}{(1 - \varepsilon_{cr}^2/2) (1 - \varepsilon_{al}^2)} + \sum_r (1 - 2\sigma_{0r}^2 + 3\sigma_{0r}^2) \alpha_r
\]

\[
\times (1 - 2\sigma_{0l}^2 + 3\sigma_{0l}^2) (1 + 2\sigma_{0l}^2 - 3\sigma_{0l}^2)
\]

\[
\times (1 - \varepsilon_{cr}^2)(1 - \varepsilon_{al}^2).
\]  

E. Sensitivity Function

The capacity sensitivity function [6] is defined as a relative capacity loss when capacities with and without system impairments are compared according to the following formula

\[
\mathcal{R} = \frac{K_{max} - K}{K_{max}}
\]  

(13)

where \(K_{max}\) is the maximum capacity without any estimation errors or impairments and \(K\) is the real capacity where channel and other estimation errors (rake efficiency, imperfect interference cancellation) are taken into account. Capacity sensitivity \(\mathcal{R} \in [0,1]\) where the lower bound \(\mathcal{R} = 0\) refers to complete insensitivity to errors (no capacity loss) and the upper bound \(\mathcal{R} = 1\) refers to very high sensitivity (all capacity lost).

IV. NUMERICAL RESULTS

Some numerical results are obtained in two 4-path channels according to Table I. Processing gain \(G = 256\) (24 dB), the required signal-to-noise ratio has been set to \(Y_0 = 2\) (3 dB), bit rate \(R_b = 16\) kbit/s and chip rate \(R_c = 4.096\) Mcchip/s. BPSK modulation is assumed. For all illustrations signal-to-noise ratio in the first rake finger is normalized to 3.2 (5 dB) in the pure background noise. Interchannel interference factor \(\nu = 0.55\). Nominal crosscorrelations are approximated to be \(p = 1/\sqrt{G}\). In the case of iterative IC the parameter estimation is made prior to interference cancellation in each channel. Phase- and delay locked loop based channel estimation is assumed. Carrier phase tracking and code delay estimation error variances are assumed to follow

\[
\sigma_{0j}^2 = 1/SNR_L,
\]

where the loop signal-to-noise ratio

\[
SNR_L = \alpha / (\alpha _A) f_d T
\]

Noncoherent amplitude estimation error [9] follows roughly \(\varepsilon_A = 1/4SNR_L\).

In Figs. 2-3 capacities and sensitivities for a conventional rake receiver have been plotted. The capacity increases when more rake fingers are allocated according to maximal ratio combining at both channel profiles. For EGC and \(\lambda = 1\) there is a capacity loss if more than two strongest paths are combined. Sensitivity increases rapidly as normalized Doppler spread \(f_d T\) increases.

Capacities and sensitivities with nonlinear interference canceling are shown in Figs. 4-5. Maximum capacities with one or two rake fingers are clearly higher than with conventional receivers. For \(\lambda = 1\) the capacity decreases for multiple rake fingers. This observation indicates that the interference canceling in the weak paths actually sums up interference rather than suppresses it.

Figs. 6-7 illustrate capacities and sensitivities of a system employing a linear decorrelating detector. This receiver structure is able to cancel all intracell interference at the cost of noise enhancement. This explains high nominal capacities.
However, the capacity decreases significantly when estimation errors are included. For higher crosscorrelations between users the noise enhancement rises and thus reduces capacity.

Fig. 2. Capacities (a) and sensitivities (b) with a matched filter rake only ($\lambda = 0.5$).

Fig. 3. Capacities (a) and sensitivities (b) with a matched filter rake only ($\lambda = 1$).

Fig. 4. Capacities (a) and sensitivities (b) with a nonlinear interference canceler ($\lambda = 0.5$).

Fig. 5. Capacities (a) and sensitivities (b) with a nonlinear interference canceler ($\lambda = 1$).

Fig. 6. Capacities (a) and sensitivities (b) with a linear decorrelator ($\lambda = 0.5$).

Fig. 7. Capacities (a) and sensitivities (b) with a linear decorrelator ($\lambda = 1$).

V. CONCLUSIONS

Capacities of wideband CDMA networks employing either conventional rake receivers or multiuser detectors were compared. Furthermore, their sensitivities to estimation errors, depending on the normalized Doppler and multipath intensity profile, were studied. As expected, the multiuser detectors could give significant capacity gains over single-user receivers. However, in severe fading the gains might be lost due to the high sensitivity to estimation errors. Linear decorrelator gave highest capacity estimates but its performance would degrade rapidly if users’ crosscorrelations would be higher.

APPENDIX

The individual elements in the estimated crosscorrelation matrix $\hat{R} = R + \Delta R$ can be presented as

$$\hat{R} = \bar{R} \cos \hat{\epsilon}.$$  \hspace{1cm} (A1)

Let the estimated phase difference $\hat{\epsilon}$ between users 1 and 2 be represented as

$$\hat{\epsilon} = \phi_1 - \Delta \phi_1 - \phi_2 - \Delta \phi_2 = \epsilon + \Delta \epsilon$$ \hspace{1cm} (A2)

where $\epsilon = \phi_1 - \phi_2$ and $\Delta \epsilon = (\Delta \phi_1 + \Delta \phi_2)$. If $\Delta \phi$ is a zero mean Gaussian process with variance $\sigma_\phi^2$, then $\Delta \epsilon$ is a zero mean Gaussian process with variance $2\sigma_\phi^2$. The estimated correlation function can be represented as
\[
\hat{\rho} = \rho + \Delta \rho = \rho + \rho' \varepsilon \tau = \rho \left(1 + \frac{\rho' \varepsilon \tau}{\rho}\right) = \rho (1 + s' \rho) \tag{A3}
\]

where \(\rho'\) is the slope of the correlation function at the point of zero delay estimation error and

\[
\varepsilon \tau = \Delta \tau_1 - \Delta \tau_2 \tag{A4}
\]

is the difference between the two delay estimation errors. If \(\Delta \tau\) is a zero mean Gaussian variable with variance \(\sigma_{\tau}^2\), then \(\varepsilon \tau\) is a zero mean Gaussian variable with variance \(2\sigma_{\tau}^2\). The second component of (A1) can be represented as

\[
\cos \hat{\varepsilon} = \cos(\varepsilon + \Delta \varepsilon) = \cos \varepsilon \cos \Delta \varepsilon - \sin \varepsilon \sin \Delta \varepsilon
\]

\[
= \left(1 - \Delta \varepsilon^2/2\right) \cos \varepsilon - \Delta \varepsilon \sin \varepsilon \left(1 + s_{\varepsilon} \cos \varepsilon\right) \tag{A5}
\]

where

\[
s_{\varepsilon} = - \left(\Delta \varepsilon^2/2 + \Delta \varepsilon \tan \varepsilon\right). \tag{A6}
\]

Now, (A1) becomes

\[
\hat{R} = R + \Delta R \tag{A7}
\]

where \(R = \rho \cos \varepsilon\) and \(\Delta R = \rho \cos \varepsilon (s_{\varepsilon} + s_{\rho} + s_{\varepsilon} s_{\rho})\). Parameter \(\Delta R\) can be considered as an additional noise component with zero mean distribution and variance

\[
\sigma_{\Delta R}^2 = \rho^2 \left[\left(1 + 2\sigma_{\varepsilon}^2 / \rho^2\right) \left(3\sigma_{\rho}^4 + 2\sigma_{\rho}^2 \sigma_{\varepsilon}^2 / \rho^2\right) + 2\sigma_{\varepsilon}^2 \sigma_{\rho}^2 / \rho^2\right] \tag{A8}
\]

where \(\sigma_{\rho}^2\) is the variance of the correlation function slope at the point of zero delay estimation error.

REFERENCES


