A New Method for Affine Registration of Images and Point Sets

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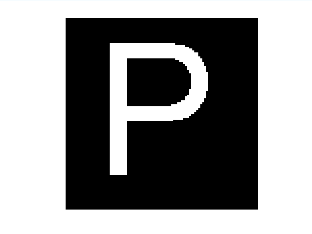
Introduction

- Task: to recover the affine transformation between two images or point sets



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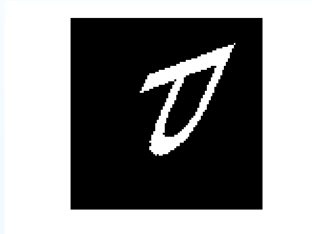

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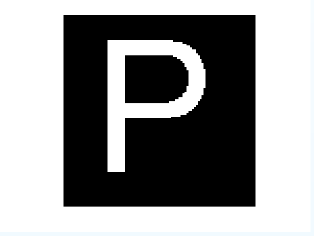

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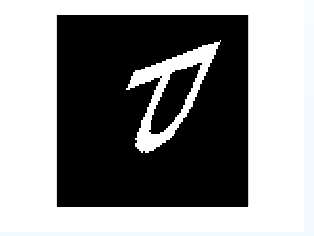

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- Possible problems:
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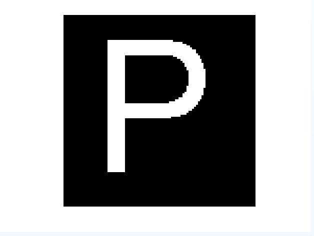

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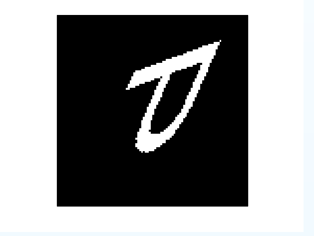

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- Possible problems:
  - sometimes it may be difficult to find enough features that can be localized accurately and matched reliably
  - when matching point sets it is an essential part of the problem that correspondences are unknown a-priori
- We propose a different approach that does not require separate feature extraction or correspondence search.

Background

Our registration method is based on a novel affine invariant image transform that was recently proposed for computation of global affine invariant features from grayscale images. (Rahtu *et al.*: A new efficient method for producing global affine invariants, ICIAP 2005.)

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Let $f : \mathbb{R}^2 \rightarrow \mathbb{R}$ be an image function, whose centroid is $\boldsymbol{\mu}(f)$.
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$$\tilde{f}(\mathbf{x}) = f(\mathbf{x} + \boldsymbol{\mu}(f))$$

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Definition 1 *For $\alpha, \beta \in \mathbb{R}$ the transform If of f is defined by*

$$If(\alpha, \beta) = \frac{1}{\|f\|_{L^1}} \int_{\mathbb{R}^2} \tilde{f}(\mathbf{x})\tilde{f}(\alpha\mathbf{x})\tilde{f}(\beta\mathbf{x})\, d\mathbf{x}.$$

Proposition 1 $I(f \circ \mathcal{A}^{-1}) = If$ *for any affine transformation $\mathcal{A}$.*

Registration Method

A slight modification of the previous formula leads to a different transform that can be used in registration.

Instead of $If(\alpha, \beta) \in \mathbb{R}$ we compute transform values in $\mathbb{R}^2$:

Definition 2 *Let* $f \in L^\infty(\mathbb{R}^2)$ *be compactly supported. For* $\alpha, \beta \in \mathbb{R}$ *define*

$$\mathbf{J}f(\alpha, \beta) = \frac{1}{\|f\|_{L^1}} \int_{\mathbb{R}^2} \mathbf{x}\tilde{f}(\mathbf{x})\tilde{f}(\alpha\mathbf{x})\tilde{f}(\beta\mathbf{x})\, d\mathbf{x}.$$

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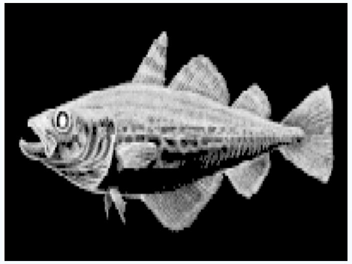
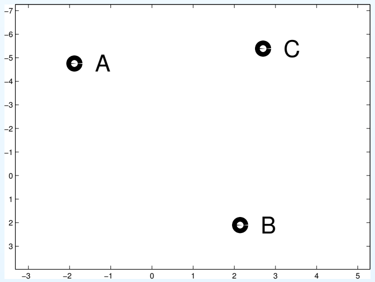
Proposition 2 *Let $\mathcal{A}(\mathbf{x}) = \mathbf{A}\mathbf{x} + \mathbf{t}$ be an affine transformation. Let f' be the $\mathcal{A}$ transformed version of f, i.e., $f'(\mathbf{x}) = (f \circ \mathcal{A}^{-1})(\mathbf{x})$. Then $\mathbf{J}f' = \mathbf{A}\mathbf{J}f$.*

By computing the transform values $\mathbf{J}f'(\alpha_i, \beta_i)$ and $\mathbf{J}f(\alpha_i, \beta_i)$ for at least two different pairs (α_i, β_i) one obtains a set of linear equations from which the transformation matrix $\mathbf{A}$ may be solved. Thereafter the translation may be solved by $\mathbf{t} = \boldsymbol{\mu}(f') - \mathbf{A}\boldsymbol{\mu}(f)$.

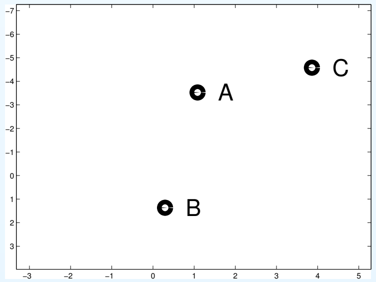
Example




Given an image f and its affine transformed version f' compute the transform values $\mathbf{J}f$ and $\mathbf{J}f'$ at points (α_i, β_i). An example with three points, $(\alpha_i, \beta_i) = \{\ (1,1),\ (-1,-1),\ (-1,1/2)\ \}$.



$$\mathbf{J}f \quad \xrightarrow{\mathbf{A}} \quad \mathbf{J}f'$$



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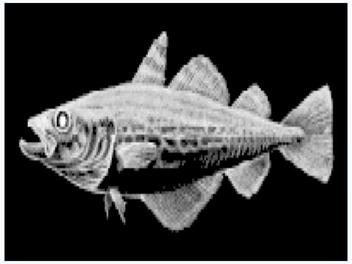

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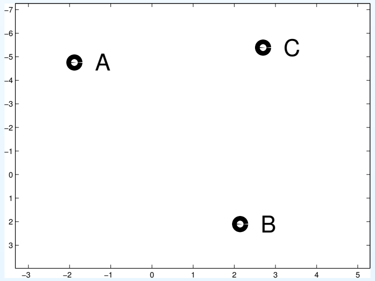

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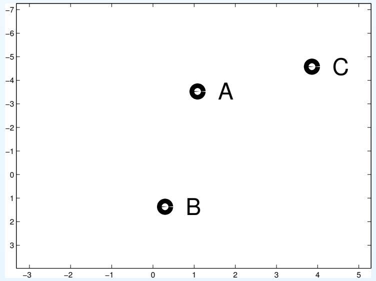
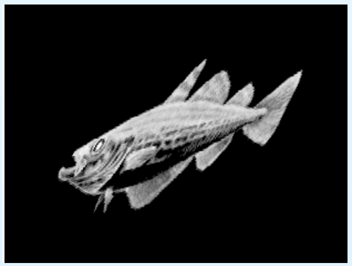


The recovered transformation $\mathbf{A}$:



Implementation and Symmetries

The discrete version of the transform

$$\mathbf{J}f(\alpha_i, \beta_i) = \frac{1}{\sum_k f(\mathbf{x}_k)} \sum_k \mathbf{x}_k \tilde{f}(\mathbf{x}_k) \tilde{f}(\alpha_i \mathbf{x}_k) \tilde{f}(\beta_i \mathbf{x}_k).$$

How one should choose the points (α_i, β_i)?

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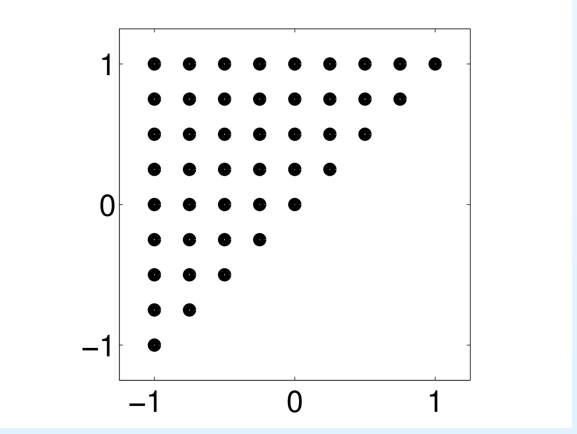
How one should choose the points (α_i, β_i)?

Proposition 3 (a) $\mathbf{J}f(\alpha, \beta) = \mathbf{J}f(\beta, \alpha)$,

(b) $\mathbf{J}f(\alpha, \beta) = \alpha^{-3}\mathbf{J}f(1/\alpha, \beta/\alpha)$ *for* $\alpha \neq 0$,

(c) $\mathbf{J}f(\alpha, \beta) = \beta^{-3}\mathbf{J}f(1/\beta, \alpha/\beta)$ *for* $\beta \neq 0$.

$\Rightarrow$ One should take points from the triangle $\{(-1,-1), (-1,1), (1,1)\}$.



Registration of Point Sets

The method must be modified before it can be applied to point sets. The role of the intensity functions is given to the probability density functions that determine the spatial distribution of points in the sets.

Two point patterns: $\mathbf{x}_j$ and $\mathbf{x}'_j = \mathbf{A}\mathbf{x}_j + \mathbf{t}$

Assume that points $\mathbf{x}_j$ and $\mathbf{x}'_j$ are samples of random variables $\mathbf{X}$ and $\mathbf{X}'$, which have means $\boldsymbol{\mu}$, $\boldsymbol{\mu}'$ and covariances $\mathbf{C}$, $\mathbf{C}'$, respectively.

The corresponding mean-zero variables are defined by $\tilde{\mathbf{X}} = \mathbf{X} - \boldsymbol{\mu}$ and $\tilde{\mathbf{X}}' = \mathbf{X}' - \boldsymbol{\mu}'$, and they have densities $\tilde{p}$ and $\tilde{p}'$.

Registration of Point Sets (continued)

We introduce an additional function

$$g(\mathbf{x}) = N(\mathbf{0}, \mathbf{C}),$$

where $N(\mathbf{0}, \mathbf{C})$ is the zero-mean Gaussian with covariance $\mathbf{C}$.

For $\gamma \in \mathbb{R}$ we define descriptor $\mathbf{H}(\gamma) \in \mathbb{R}^2$ as follows

$$\mathbf{H}(\gamma) = \frac{\int_{\mathbb{R}^2} \mathbf{x} g(\gamma \mathbf{x}) \tilde{p}(\mathbf{x}) d\mathbf{x}}{\int_{\mathbb{R}^2} g(\gamma \mathbf{x}) \tilde{p}(\mathbf{x}) d\mathbf{x}} = \frac{E[\tilde{\mathbf{X}} g(\gamma \tilde{\mathbf{X}})]}{E[g(\gamma \tilde{\mathbf{X}})]},$$

and $\mathbf{H}'(\gamma)$ is defined similarly for the second pattern.

It can be shown that again it holds

$$\mathbf{H}'(\gamma) = \mathbf{A}\mathbf{H}(\gamma).$$

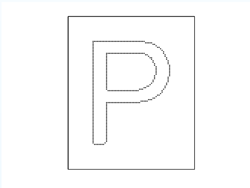
Summary of Point Pattern Matching

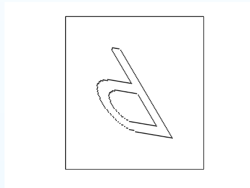
1. Given two sets of points $\{\mathbf{x}_j\}$ and $\{\mathbf{x}'_j\}$ compute the corresponding sample means, $\bar{\mathbf{x}}$ and $\bar{\mathbf{x}}'$, and sample covariances, $\mathbf{C}$ and $\mathbf{C}'$.
2. Compute the normalized points $\tilde{\mathbf{x}}_j = \mathbf{x}_j - \bar{\mathbf{x}}$ and $\tilde{\mathbf{x}}'_j = \mathbf{x}'_j - \bar{\mathbf{x}}'$. Set $g = N(\mathbf{0}, \mathbf{C})$ and $g' = N(\mathbf{0}, \mathbf{C}')$.
3. Compute the points $\mathbf{H}(\gamma_i)$ and $\mathbf{H}'(\gamma_i)$. The expectation values are computed as sample means by using the samples $\tilde{\mathbf{x}}_j$ and $\tilde{\mathbf{x}}'_j$.
4. Solve the affine transformation matrix $\mathbf{A}$ from point correspondences $\mathbf{H}'(\gamma_i) \leftrightarrow \mathbf{H}(\gamma_i)$ by least-squares.
5. Solve $\mathbf{t} = \bar{\mathbf{x}}' - \mathbf{A}\bar{\mathbf{x}}$.

In the experiments we used values $\gamma_i = \{0, \frac{1}{4}, \frac{1}{2}, \frac{3}{4}, 1\}$.

Experiments - Point Sets

The point pattern below (left) was transformed with random affine transformations and noise was added (center). Recovered transformation is shown on the right.





The average values of the registration error at different levels of noise:

noise	0	0.02	0.04	0.06	0.08	0.10
H	0.00	0.04	0.09	0.13	0.18	0.23
CW	0.00	0.05	0.10	0.16	0.23	0.30

H: the proposed method

CW: cross-weighted moment method
(Yang and Cohen: Cross-Weighted Moments and Affine Invariants for Image Registration and Matching, PAMI 1999)

Experiments - Images

The binary image ’P’ was transformed with random affine transformations and binary noise was added. Recovered transformation on the right.

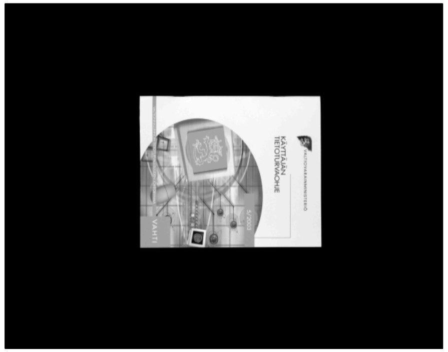
The average values of the registration error at different levels of noise:

noise	0	0.02	0.04	0.06	0.08	0.10
CW	0.11	0.17	0.21	0.28	0.29	0.34
J	0.33	0.37	0.39	0.42	0.43	0.45
H	0.09	0.14	0.18	0.22	0.26	0.29

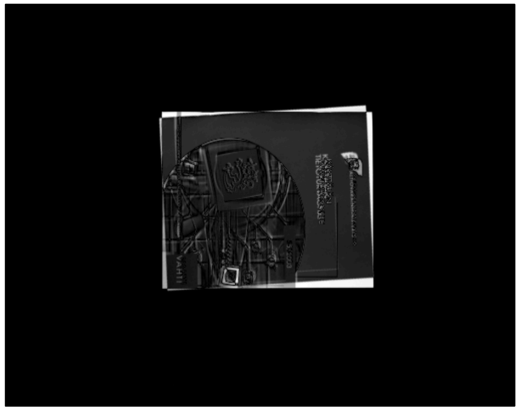
J: the proposed intensity-based method
H: the proposed point-based method (the white pixels were considered as 2D points)

Experiments - Real Photograph

Two photographs of a book were taken from different viewpoints. The dark background was segmented out and set to zero.



The registration result:



Conclusions

- a novel method for affine registration of images and point sets
- non-iterative and efficient
- utilization of the entire intensity information implies limitations (non-uniform background and occlusion cause problems)
- the method might be used to compute an initial registration which is then refined with other methods