Affine Registration with Multiscale Autoconvolution

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Abstract

- We propose a novel method for the affine registration of images and point sets.
- The approach is based on the multiscale autoconvolution transform [1].
- No separate feature extraction, intensity distribution of images or spatial distribution of points is directly utilized.
- In the experiments the proposed method performed robustly when compared to other similar methods, [2] and [3].

Multiscale Autoconvolution (MSA)

Let f be an image intensity function and $p = f/||f||_{L^1}$ the corresponding probability density function.
Let $\mathbf{X}_0, \mathbf{X}_1$, and $\mathbf{X}_2$ be independent and identically distributed random variables with the density p.
For $\alpha, \beta \in \mathbb{R}$, we set $\gamma = 1 - \alpha - \beta$ and define a random variable $\mathbf{U}_{\alpha,\beta} = \alpha\mathbf{X}_1 + \beta\mathbf{X}_2 + \gamma\mathbf{X}_0$.
The MSA transform of f is defined as the expectation value of $f(\mathbf{U}_{\alpha,\beta})$, i.e., $F(\alpha,\beta) = E[f(\mathbf{U}_{\alpha,\beta})]$.
The transform values are invariant to affine transformations of the image.

Affine Registration

Definition 1 *The MSA descriptor* $\mathbf{H}(\alpha,\beta)$ *of* f *is defined as*

$$\mathbf{H}(\alpha,\beta) = E[\mathbf{U}_{\alpha,\beta} f(\mathbf{U}_{\alpha,\beta})] \tag{1}$$

Property 1 *Let* $\mathcal{A}(\mathbf{x}) = \mathbf{T}\mathbf{x} + \mathbf{t}$ *be an affine transformation, where* $\mathbf{T}$ *is a nonsingular matrix. Let* f *be an image intensity function and* f' *the* $\mathcal{A}$ *transformed version of* f*, i.e.,* $f'(\mathbf{x}) = f(\mathcal{A}^{-1}(\mathbf{x}))$*. Then the MSA descriptor* $\mathbf{H}'(\alpha,\beta)$ *of* f' *is obtained from the corresponding descriptor of* f *by*

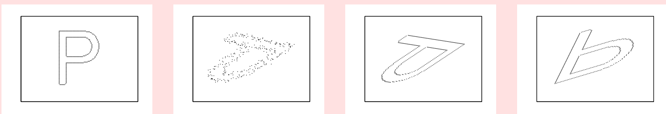
$$\mathbf{H}'(\alpha,\beta) = \mathbf{T}\mathbf{H}(\alpha,\beta) + \mathbf{t}F(\alpha,\beta). \tag{2}$$

By computing $\mathbf{H}(\alpha_i,\beta_i)$, $\mathbf{H}'(\alpha_i,\beta_i)$ and $F(\alpha_i,\beta_i)$ for at least three different pairs (α_i,β_i) one obtains a set of linear equations from which $\mathbf{T}$ and $\mathbf{t}$ may be solved.

For point sets the density p is the spatial distribution function and f in (1) is replaced by Gaussian distribution. Implementation details are described in the paper.

Experiments

The method was experimented with point patterns and binary and grayscale images. Typically, approximately correct transformation is recovered despite of noise. Since the method is global it should be used for isolated objects.



Pattern matching: original (a), transformed and noise added (b), recovered transformation with MSAP (c), and with MD (d).

Image registration: original (a), transformed and noise added (noise=0.08) (b), recovered transformation ($\varepsilon = 0.13$) (c).

The average values of the registration error ε at six levels of noise:

point set

noise	0	0.02	0.04	0.06	0.08	0.10
MD [2]	0.00	0.01	0.07	0.14	0.24	0.31
CW [3]	0.00	0.05	0.10	0.16	0.23	0.30
MSAP	0.00	0.04	0.08	0.12	0.16	0.22

binary image

noise	0	0.02	0.04	0.06	0.08	0.10
MD [2]	0.47	0.62	0.67	0.74	0.74	0.79
CW [3]	0.11	0.17	0.21	0.28	0.29	0.34
MSAD	0.05	0.09	0.13	0.15	0.16	0.20

References

[1] Rahtu, E., Salo, M., Heikkilä, J.: Affine invariant pattern recognition using multiscale autoconvolution. *TPAMI*, 27(6):908-918, 2005.
[2] Heikkilä, J.: Pattern matching with affine moment descriptors. *Pattern Recognition*, 37(9):1825-1834,2004.
[3] Yang, Z., Cohen, F.: Cross-weighted moments and affine invariants for image registration and matching. *TPAMI*, 21(8):804-814, 1999.