Abstract—Efficient decision fusion technique for cooperative spectrum sensing based on the logical OR-rule is considered for dynamic spectrum sharing between primary users (PUs) and secondary users (SUs). Due to the use of the OR-rule, the fusion center (FC) only needs to know if any of the local decisions was "1". Therefore, it can be a waste of resources to communicate each local decision in an orthogonal channel. To reduce the resource consumption, we consider that the SUs that had local decision "1" transmit simultaneously in the same channel using continuous wave (CW) signaling without a phase synchronization and that the FC will declare the PU is present if it detects the summed CWs. In this signaling, even with very large channel gain to noise power ratio (CNR) between the SUs and the FC the system may not reach error-free signaling due to destructive interference in the summed CWs. To address this problem, we propose power allocation to reduce the effects of the destructive interference, as with this technique it is possible to guarantee that the CWs are not going to fully cancel each other. The proposed method is very simple and although it is suboptimal, it could provide good detection performance. Numerical results show that it is possible to practically reach the upper bound with error-free signaling by the proposal, even with moderate CNR values.

I. INTRODUCTION

To solve the spectrum scarcity problem, dynamic spectrum access (DSA) with cognitive radio techniques between primary users (PUs) and secondary users (SUs) is one of the promising approaches for utilizing the spectrum more effectively [1], [2]. In the DSA, SU can opportunistically access the spectrum white-spaces, as long as the SU would not cause harmful interference to the operations of the PUs. For ensuring the protection of the PUs, reliable spectrum sensing is one of the key techniques in the DSA [3].

One of the significant problems in spectrum sensing is that reliable spectrum sensing is difficult due to multipath fading and/or shadowing. To solve this problem, cooperative spectrum sensing (CSS), which could exploit space diversity gain, has been investigated [4]–[7]. Therein, the fusion center (FC) gathers local decisions from the SUs (either hard or soft decisions) and makes the global decision. Although using soft decisions may lead to better performance [8]–[11], using hard decisions requires less signaling overhead.

In an OR-rule based CSS, the FC only needs to know either all of the local decisions are $H_0$ corresponding to the absence of PU or at least one of the local decisions is $H_1$ corresponding to the presence of PU. It implies that in the OR-rule, minimal required resources for the signaling is $O(1)$. However, conventionally even with the logical OR-rule, each sensor is allocated orthogonal resources to transmit its local decisions [10], [12]–[14], potentially leading to a waste of resources.

In [15], [16], it has been proposed that all local decisions by the OR-rule are sent simultaneously in the same channel using non-orthogonal multiple access and non-coherent reception. It is obvious that this approach is much more effective than the conventional $O(N_S)$ approach, where $N_S$ indicate number of SUs, since the amount of resources used in signaling is $O(1)$. However, this signaling may not be sufficiently reliable due to destructive interferences between signals and the effect of destructive interference was not investigated.

In [17] and [18], a decision fusion in distributed network such as wireless sensor network is investigated. In [17], signature vectors are used to reduce the effect of destructive interference. In [18], it was shown that diversity technique can reduce the effect of destructive interference under non-orthogonal multiple access and non-coherent reception case.

In this paper, motivated by the aforementioned works, we investigate how the SUs can reduce the effects of the destructive interference to enable the use of $O(1)$ scaling for signaling, with higher reliability as compared to the conventional approaches. At first, we provide the exact analysis of the performance of the OR-rule with the $O(1)$ signaling. Reference [18] has already performed similar analysis for the "MAC" (multiple-access channel) with random phase channel but they applied the Gaussian approximation as opposed to our exact results. As mentioned in [18], the difference can be significant for small numbers of sensors. We also find that even with large reporting channel gain to noise power ratios (CNRs) the upper bound of performance is not always reached (due to destructive interference). Next, we propose a very simple algorithm called the PA (power allocation) in the case of the $O(1)$ signaling. The PA enables reaching the upper bound of performance unlike the conventional approach because the
amplitudes are selected in such a way that the transmitted signals never fully cancel each other no matter which set of the SUs is currently transmitting. In total, we introduce and compare four decision fusion (DF) frameworks for the CSS.

II. MODEL AND FORMULATION

A SU network consists of a FC and \( N_S \) SUs. The PU has two states represented by hypotheses \( H_0 \) (the absence of PU) and \( H_1 \) (the presence of PU). All SUs share the same PU state, but their decisions are conditionally independent. We employ the Neyman-Pearson criterion thus our purpose is to maximize the global detection probability \( P_{G,D} \) while the target false alarm probability \( \tilde{P}_{G,FA} \) is maintained [19]. The CSS based on OR-rule is composed of three processes; local observation, signaling, and final decision.

During the local observation, at the \( i \)th SU \( (i = 1, 2, \ldots, N_S) \), a local decision \( \hat{H}_{L,i} \) is obtained by an energy detector with local false alarm probability \( \tilde{P}_{L,FA,i} \) and local detection probability \( \tilde{P}_{L,Di} \). Without the loss of generality, we assume that the observed PU signal and noise at the \( i \)th local SU follow the circularly symmetric complex Gaussian distribution as \( \mathcal{CN}(0, \sigma_{PU,i}^2) \) and \( \mathcal{CN}(0, \sigma_{w}^2) \), respectively [7]. Signal-to-noise ratio at a local SU for the local observation is the output of the energy detector at the local observation, signaling, and final decision.

The signaling of the local decisions and the final decision process depend on the applied DF framework.

A. DF with perfect signaling: DF_{UP}

Although not always practical, most of the related works assume error-free signaling, e.g., [5], [8]. We call this DF_{UP}. Based on the OR rule, in the DF_{UP}, the FC decides \( \hat{H} = H_1 \), where \( \hat{H} \) denote final decision, if any (even one) of the local decisions was \( H_1 \), otherwise \( \hat{H} = H_0 \).

The target local false alarm probability \( \tilde{P}_{L,FA} \) has to satisfy the following equality [5]

\[
\tilde{P}_{G,FA} = 1 - (1 - \tilde{P}_{L,FA})^{N_S}.
\]

In DF_{UP}, local energy detector threshold and \( \gamma_{L,i} \) determine \( P_{L,D,i} \). Finally, the global detection probability in the DF_{UP} is given

\[
P_{G,D} = 1 - \prod_{i=1}^{N_S} (1 - P_{L,D,i}),
\]

and this is the upper bound of the CSS based on the OR-rule.

B. DFs with non imperfect signaling

A censoring is implemented by the on-off keying [18]. Specifically, only the SUs which have a local decision \( \hat{H}_{L,i} = H_1 \) transmit a continuous wave (CW) toward the FC in a dedicated control channel and the other SUs do not transmit any signal. The transmitting signal at the \( k \)th instant \((k = 0, 1, \ldots, K-1)\) by the \( i \)th SU is formulated as follows;

\[
x_i(k) = \begin{cases} 0 & (\hat{H}_{L,i} = H_0) \\ A_i \exp(j\omega_c k) & (\hat{H}_{L,i} = H_1), \end{cases}
\]

where \( A_i \) is the amplitude component and \( \omega_c \) is the normalized carrier frequency. The amplitude \( A_i \) can be allocated by the PA otherwise \( A_i = 1 \). The duration of the time slot for the signaling corresponds to the \( K \) samples. The received signal of the FC at the \( k \)th instant is as follows;

\[
y(k) = \sum_{i \in I_1} h_i x_i(k) + w(k) = s(k) + w(k),
\]

where \( I_1 \) is the set of the SUs which have a local decision \( \hat{H}_{L,i} = H_1 \), \( h_i \) is the channel coefficient between the \( i \)th SU and the FC, \( w(k) \) is a zero-mean complex additive white Gaussian noise (AWGN), i.e., \( \mathcal{CN}(0, \sigma_w^2) \) where \( \sigma_w^2 \) denotes the noise variance of the FC, and \( s(k) \) is the signal component of the received signal. The channel coefficient is represented by \( h_i = \alpha_i \exp(j\phi_i) \) where \( \alpha_i \) is the amplitude component and \( \phi_i \) is the phase component. Without the loss of generality we assume that \( \alpha_i \geq \alpha_{i-1} \). It is also assumed that the channel coherence time is much larger than process of the CSS thus information of the channel gain \( \alpha_i \) used in the PA is available at the FC and each SU. Due to non-coherent reception at the FC, \( \phi_i \) are uniformly distributed in a region where \( 0 < \phi_i \leq 2\pi \) and independent and identically distributed. Therefore, these channels could be treated as random phase channel and signaling can be distorted by destructive interference [18].

At the FC, an CW signal detector is employed and the output of the detector, so-called test statistic, is given by [19]

\[
T = \frac{1}{K} \left| \sum_{k=0}^{K-1} y(k) \exp(-j\omega_c k) \right|^2.
\]

The CNR in the link between the FC and the \( i \)th SU is defined by \( \gamma_{FC,i} = (K\alpha_i^2)/\sigma_w^2 \) where \( K \) acts as a scaling factor. The FC makes the final decision \( \hat{H} = H_1 \) if \( T > \tau_G \) otherwise the FC decides \( \hat{H} = H_0 \). The threshold \( \tau_G \) is chosen to satisfy the target false alarm probability as

\[
\tilde{P}_{G,FA} = \int_{T>\tau_G} p(T|H_0)dT,
\]
where $p(T|H_0)$ is a conditional PDF of $T$ for the hypothesis $H_0$. This decision rule employs the same concept as the OR-rule in the DF$_{T,P}$. The inequality $T > \tau_G$ implies that one or more than one local decision is $H_1$. Otherwise, $H = H_0$ since all of the local decisions may be $H_0$.

III. DECISION FUSION WITHOUT THE PA (DF$_n$)

In the DF$_n$, the PA is not used thus $A_i = 1$.

A. Analysis

At first we analyze the DF$_n$ in terms of the probability density function (PDF) of $T$ and detection probability. The signal component in $T$ is given by

$$S = \frac{1}{K} \sum_{k=0}^{K-1} s(k) \exp(-j\omega_k)|^2$$

(2)

where $\beta_i = \alpha_i A_i \sqrt{K}$. The PDF of $S$ is equivalent to the PDF of the squared envelope of a sum of random phase vectors with arbitrary amplitudes. A recursive solution proposed in [20] to calculate the PDF of $S(I_1)$ where $I_1 = \{i_1, i_2, \ldots, i_N\}$ is introduced as follows. At first, given $I_1 = \{i_1, i_2\}$, the PDF of $S$ redenoted by $S_2$ is

$$p_2(S_2) = \begin{cases} \frac{1}{\pi} \sum_{i=1}^{K} KA_i \alpha_i \exp(j\phi_i)^2 = 1 \sum_{i=1}^{K} \beta_i \exp(j\phi_i)^2, \\ (\beta_{i1} - \beta_{i2})^2 \leq S_2 \leq (\beta_{i1} + \beta_{i2})^2, \\ 0; \quad \text{(otherwise)}. \end{cases}$$

(4)

Given $I_1 = \{i_1, i_2, \ldots, i_n\} \quad 2 < n \leq N_1$, the PDF $S_n$ can be recursively obtained by

$$p_n(S_n) = \begin{cases} \frac{1}{\pi} \sum_{i=1}^{K} P_{n-1}(S_{n-1}) \sum_{i=1}^{K} \beta_i \exp(j\phi_i)^2, \\ (\beta_{i1} - \beta_{i2})^2 \leq S_n \leq (\beta_{i1} + \beta_{i2})^2, \\ 0; \quad \text{(otherwise)} \end{cases}$$

(5)

where

$$\xi_{n-1} = \min_{k=1}^{K} (\sum_{k=1}^{K} \beta_{i_k})^2, \quad (\beta_{i_n} + \sqrt{S_n})^2$$

and

$$\delta_{n-1} = \max[S_{n-1,\text{min}}, (\beta_{i_n} - \sqrt{S_n})^2]$$

and

$$S_{n,\text{min}} = \begin{cases} (\beta_{i_n} - \sum_{k=1}^{K} \beta_{i_k})^2; \quad \beta_{i_n} \geq \sum_{k=1}^{K} \beta_{i_k}, \\ (\sum_{k=1}^{K} \beta_{i_k})^2; \quad \beta_{i_n} \leq \sum_{k=1}^{K} \beta_{i_k}. \end{cases}$$

(6)

Given $I_1 = \{i_1, i_2, \ldots, i_N\}$ and $H_1$, the conditional PDF of the $T$ follows the noncentral $\chi^2$-square distribution with the signal component $S_{N_1}$ corresponding to the noncentrality parameter. By marginalizing over $S_{N_1}$ in the conditional PDF, $p(T|I_1, H_1)$ can be obtained as

$$p(T|I_1, H_1) = \begin{cases} \int_{S_{N_1}=S_{N_1,\text{max}}}^{S_{N_1,\text{max}}} p_{n_n}(T|S_{N_1}) \, dS_{N_1}; \\ p_{n_n}(S_{N_1}) \, dS_{N_1}; \quad T \geq 0, \\ 0; \quad T \leq 0, \end{cases}$$

(7)

where $S_{N_1,\text{max}} = (\sum_{i=1}^{K} \beta_{i_k})^2$ and $p_{n_n}(x)$ is the noncentral $\chi^2$-square distribution with 2 degrees of freedom and noncentrality parameter $y$, so that the probability density function is

$$p_{n_n}(x) = \frac{1}{2} \exp\left(-\frac{1}{2}(x + y)\right) I_0(\sqrt{xy}),$$

where $I_0$ is the modified Bessel function of the first kind and order 0. In [18], $T$ in $(T|I_1, H_1)$ is assumed to follow the noncentral $\chi^2$ distribution according to the Gaussian approximation. However, this approximation is valid for large $N_1$ as mentioned in [18].

The probability $Pr(I_1|H_i)$ is given by

$$Pr(I_1|H_i) = \begin{cases} \frac{n}{\pi} \sum_{i=1}^{K} P_{n-1}(S_{n-1}) \sum_{i=1}^{K} \beta_i \exp(j\phi_i)^2; \\ (\beta_{i1} - \beta_{i2})^2 \leq S_n \leq (\beta_{i1} + \beta_{i2})^2, \\ 0; \quad \text{(otherwise)}. \end{cases}$$

(8)

Finally, based on the result in (8), the global detection probability at the FC is given by

$$P_{G,D} = \int_{T \geq \tau_G} p(T|H_1) \, dT$$

where $\tau_G$ is set to satisfy the target $P_{G,F,A}$ as

$$P_{G,F,A} = \int_{T \geq \tau_G} \int_{T \geq \tau_G} p(T|H_0) \, dT.$$

The global detection probability, $P_{G,D}$, indicates the probability that envelope detector output at the FC exceeds the threshold for global decision $\tau_G$.

IV. POWER ALLOCATION ALGORITHM IN DF$_P$

A. Algorithm

In this proposed framework, we assume that the FC can allocate the transmit amplitudes of the SUs, $A = [A_1, \ldots, A_{N_S}]$, with the aim to maximize the global detection probability. The optimization problem is now

$$A_{opt} = \arg \max_A \int_{T \geq \tau_G} \sum_{i=1}^{N_S} p(T, A|I_1, H_1) \Pr(I_1|H_1) \, dT$$

subject to a transmit power constraint

$$\sum_{i=1}^{N_S} A_i^2 = N_S,$$
where $A_{opt}$ is the optimal solution. The above optimization problem requires knowledge of the local detection probabilities in $Pr(I_1|H_1)$. It is difficult to obtain $Pr(I_1|H_1)$ at the secondary user network. Even if $Pr(I_1|H_1)$ is available, the derivation of $A_{opt}$ is not easy.

Therefore, we study a more practical approach with a sub-optimization problem where the goal is the maximization of $S_{min}$. The sub-optimization problem is given by

$$A_{sub-opt} = \arg \max_A \min S_{min}$$

subject to

$$\sum_{i=1}^{N_S} A_i^2 = N_S,$$

where $A_{sub-opt}$ is the optimal solution for the above optimization problem and $S_{min} = \{S_{min}|I_1\in I_{all} \setminus \emptyset\}$. $S_{min}(I_1)$ indicates the minimum $S$ under given $I_1$ as shown in (6). The aim of this optimization is to increase the minimum of $S_{min}$.

To understand the intuitive meaning of the sub-optimization problem, an example where $N_S = 2$, $A_1 = A_2$ and $\alpha_1 = \alpha_2$ for all $i$ is introduced as follows. According to (6), in this example, $S_{min}$ is given by

$$S_{min} = \{S_{min}(\{1\}), S_{min}(\{2\}), S_{min}(\{1, 2\})\} = \{\beta_1^2, \beta_2^2, (\beta_1 - \beta_2)^2\} = \{\beta_1^2, \beta_2^2, 0\}.$$  

In this example, the worst case is $0$ in $(\beta_1 - \beta_2)^2$ due to destructive interference between the signals by two SUs (with $\pi$ phase difference). On the other hand, this result implies that $PA$ can increase the term $(\beta_1 - \beta_2)^2$ for example by setting $A_1 < A_2$ because in this case the components cannot fully cancel each other no matter what are their phases. However, it leads to decreasing the other term $\beta_1^2$ due to the transmit power constrain. In this case, the optimum solution can be achieved by $(\beta_1 - \beta_2)^2 = \beta_2^2$.

Now the general solution for (10) is introduced as follows. The optimized solution satisfies the following condition for $0 < i < N_S$;

$$\beta_i = \frac{\beta_{i+1}}{2}.$$  

Consequently, the optimum amplitude for $i$th SU is given by

$$A_{sub-opt,i} = \frac{2^{i-1} \alpha_1}{\alpha_i} A_{sub-opt,1},$$

where

$$A_{sub-opt,1} = \sqrt{\frac{N_S}{\sum_{i=1}^{N_S} 4^{i-1} \alpha_i^2}}$$

according to the transmit power constraint (11). The obtained $\min(S_{min})$ under the optimum solution is bounded by $\beta_1^2$, specifically

$$\min(S_{min}) = (\beta_n - \sum_{k=1}^{n-1} \beta_k)^2 = \beta_1^2 = \frac{N_S K}{\sum_{i=1}^{N_S} 4^{i-1} \alpha_i^2},$$

where $2 \leq n \leq N_S$. The analysis in section III-A is also valid in the case of $DF_P$ since arbitrary $A_i$ was considered therein.

### B. Numerical results (DF$_P$)

To confirm the benefit of the PA, the performances of $DF_P$ are compared against the $DF_n$, the $DF_{UP}$ and the $DF_O$ which employs the PA with full-search optimization to achieve the optimum solution $A_{opt}$ in (9), in Figs 1 and 2.

In Fig. 1, $P_{G,D}$ as a function of $\gamma_{FC}$ dB is shown in the cases of $N_S = 2$, $\gamma_L = -5$ dB and $-3$ dB. The target $P_{G,D}$ is set as $0.9$. In terms of the required CNR for the target $P_{G,D}$, the $DF_P$ has more than 7 dB gain and 9 dB gain in the cases $\gamma_L = -5$ dB and $-3$ dB respectively against the $DF_n$. Differences about the required CNR for the target $P_{G,D}$ between the $DF_P$ and the $DF_O$ are less than 1 dB and less than 3.5 dB in the cases $\gamma_L = -5$ dB and $-3$ dB respectively. One of the notable results is that $DF_P$ and $DF_O$ attain the upper bound $P_{G,D}$ at the approximately-same CNR although the $DF_n$ did not attain it even at $\gamma_{FC} = 20$ dB. In the $DF_n$, $\min(S_{min}) = 0$ but in the $DF_P$, $\min(S_{min}) = \beta_1^2$. If $\beta_1^2 \gg \tau_O$, the $P_{G,D}$ performance of the $DF_P$ can reach the upper bound.

In Fig. 2, the same numerical evaluation as in Fig. 1 but now $N_S = 3$ is shown. At the target $P_{G,D}$, the $DF_P$ has 3.5 dB gain and 4.5 dB gain in the cases $\gamma_L = -5$ dB and $-3$ dB respectively against the $DF_n$. The $DF_O$ has less than 0.5 dB gain and less than 3.5 dB gain in the cases $\gamma_L = -5$ dB and $-3$ dB respectively against the $DF_P$. In the $DF_O$, local detection probabilities are considered to achieve the actual optimal solution unlike $DF_P$ and it causes the gain in $DF_O$ compared to the $DF_P$.

This fact implies that the power allocation with the local detection probabilities information has a potential to improve the detection performance of the proposed method however the knowledge of local detection probabilities at the SU side is not usually a practical approach and this matter is not our present concern.

Theoretical results for $DF_P$ in terms of $P_{G,D}$ are also plotted by circle marker in Figs. 1 and 2. It can be seen that the theoretical and simulation results agree very well.

### V. Conclusions

Efficient $DF$ with $O(1)$ scaling for signaling for a CSS based on the logical OR-rule was considered. We have shown that the detection performance of the the $DF_n$ can not attain the upper bound even in high CNR region due to destructive interference.

For this problem, we introduced the $DF_P$ where the proposed PA is employed. The PA is simple and can maximize the minimum signal component of the test statistic. Thus the detection performance can attain the upper bound in the high CNR region, specifically the maximized minimum signal component is enough larger than the global threshold. The performance difference between the proposed $DF_P$ and the $DF_O$ with the optimum PA approach is negligible, e.g., less than 1 dB in CNR. We also have confirmed that the theoretical and simulation results of $DF_P$ agree very well.
Fig. 1. Detection probabilities $P_{O,D}$ as a function of CNR ($\gamma_{FC}$) dB where $\gamma_{FC,i} = \gamma_{FC}$ for all $i$ in terms of the upper bound $DF_{UP}$ (bold line), the $DF_n$ (solid line), the $DF_F$ (dot line) employing proposed PA and the $DF_O$ with the PA employing the full-search optimization (bold dot line). $P_{C,F_A} = 0.1$ and $N_{PU} = 64$. Analysis of $DF_F$ is circle marker.

Fig. 2. Detection probabilities $P_{O,D}$ as a function of CNR ($\gamma_{FC}$) dB where $\gamma_{FC,i} = \gamma_{FC}$ for all $i$ in terms of the upper bound $DF_{UP}$ (bold line), the $DF_n$ (solid line), the $DF_F$ (dot line) employing proposed PA and the $DF_O$ with the PA employing the full-search optimization (bold dot line). $P_{C,F_A} = 0.1$ and $N_{PU} = 64$. Analysis of $DF_F$ is circle marker.

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