The Capacity of Wireless Ad Hoc Networks

Presentation for the postgraduate course "Ad Hoc Networks"

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Introduction (1/3)

- This presentation deals with:
  1) **Fixed** wireless ad hoc networks
  2) **Mobile** wireless ad hoc networks
       - Extension of [Gup00] into a case where the nodes are allowed (actually required) to move

Introduction (2/3)

- Theoretical capacity analysis give us guidelines for the fundamental limits of the wireless ad hoc networks:
  - What is the maximum per-user throughput of the network (with given constraints)?
  - How does the network perform in asymptotic conditions (for example, the number of users $n \rightarrow \infty$)?
  - How can we achieve these limits?
  - **Hints for practical design problems!**
### Introduction (3/3)

- **Big O notation:**
  1. If $f(n) = O(g(n))$ then $\exists c_2, n_0 \in \mathbb{R}^+ \text{ so that } 0 \leq f(n) \leq c_2 g(n), \forall n \geq n_0$
  2. If $f(n) = \Theta(g(n))$ then $\exists c_1, c_2, n_0 \in \mathbb{R}^+ \text{ so that } 0 \leq c_1 g(n) \leq f(n) \leq c_2 g(n), \forall n \geq n_0$

![Graph showing Big O notation](image)

### Fixed Ad Hoc Network (1/5)

- **Fixed random networks**

  Distance between a node-pair (i,j) is $d_{i,j} = |X_i - X_j|$ and the signal power decreases as $d_{i,j}^{-\alpha}, \alpha \geq 2$; only path loss is considered.

  Transmission that blocks the nearby nodes.

  There are $n$ nodes in the network.

  The area is fixed to a disk of area $1 \text{ m}^2$ (or a surface of a 3-D sphere) for notational convenience.

  All nodes can transmit at a rate of $R$.

  Nodes that are allowed to transmit.

  Nodes that are not allowed to transmit.

  Transmitting nodes $\{X_i : k \in T\}$ employ nominal transmission powers $\{P_k : k \in T\}$.
Fixed Ad Hoc Network (2/5)

• Under a "physical model", a transmission from $X_i$ to $X_j$ is successful if

$$\frac{P_i}{|X_i - X_j|^\alpha} \geq \beta; \quad (P_i = P_k = P \forall k, i \in T)$$

where $L$ is the processing gain of the system, $N_0$ the background noise power and $\beta$ the SINR threshold required by the receiver.

Fixed Ad Hoc Network (3/5)

• A throughput of $\lambda(n)$ is **feasible** if
  – There is a spatial and temporal scheduling scheme $\pi$, such that by allowing multiple hops and buffering at intermediate nodes, all nodes achieve an average rate of $\lambda(n)$
  – Random network under the physical model:

$$\exists c_1, c_2 \in \mathbb{R}^+: \begin{cases} \lim_{n \to \infty} \Pr\left\{ \lambda(n) = \frac{c_1 R}{\sqrt{n \log n}} \text{ is feasible} \right\} = 1 \\ \lim_{n \to \infty} \Pr\left\{ \lambda(n) = \frac{c_2 R}{\sqrt{n}} \text{ is feasible} \right\} = 0 \end{cases}$$
Fixed Ad Hoc Network (4/5)

• Implications of the previous result:
  – Within a factor \(1/\sqrt{\log n}\) the average achievable throughput per-node is \(O\left(R/\sqrt{n}\right)\)
    • Achieved with perfect scheduling, routing and relaying; common transmission power \(P\)
    • In practice the situation is much worse!
    – Allowing optimal traffic patterns, per-node power control \((P_i \neq P_k, i \neq k)\), scheduling, etc., the throughput is still \(\Theta\left(R/\sqrt{n}\right)\)
  
• Dividing the channel into sub-channels does not change the results

Fixed Ad Hoc Network (5/5)

• Conclusion from the results of [Gup00]:
  – What ever we do, the per-user capacity of a fixed wireless ad hoc network diminishes to zero as the “node-density” increases

• Why is the situation so pessimistic?
  – Throughput loss with common transmission range \(r(n)\) is quadratic ⇒ minimize \(r(n)\)
  – Small \(r(n)\) ⇒ large number of intermediate nodes per each “new” packet ⇒ excessive relaying decreases the per-user throughput
  – Lesson: Avoid “dense” ad hoc networks.
Mobile Ad Hoc Network (1/6)

• [Gro02]: With mobility, a constant per-user throughput when \( n \to \infty \) is possible
  – More nodes and movement, the better
  – True if the users are willing to wait (long..)
  – The delay is proportional to the speed of change and the number of nodes in network
    • Works only with delay insensitive applications
  – The users should also stay within a limited area or a large portion of packets is lost
  – Not very realistic, but gives guidelines

Mobile Ad Hoc Network (2/6)

• Assume random ad hoc network and the “physical transmission model”
  – Location of the \( i^{th} \) node at time \( t \) is \( X_i(t) \)
  – Each node has an infinite stream of data to send to its destination
    • Source-destination (S-D) association does not change even though the nodes are moving
    • Distance between node-pair \((i,j)\) at time \( t \): \( d_{i,j}(t) \)
  – Nodes are assumed to have infinite buffers also for the relay-traffic
Scheduling policy $\pi_1$ (no relaying):

- Since the interference and/or excessive relaying were limiting the throughput in previous case, allow the mobiles only to transmit directly to each other and when they are closely located.
  - Possible in mobile case, since the randomly moving nodes are expected to be close to each other from time to time.
- Unfortunately the S-D pair is close to each other only $O(1/n)$ of the time.
- Throughput per S-D pair goes to zero as $\frac{1}{n^{1+\alpha/2}}$; for example typical urban scenario: $\alpha = 4 \Rightarrow 1/\sqrt[3]{n}$.

Scheduling policy $\pi_2$ (with relaying):

1) Distribute your own packets to other nodes when one is in an immediate vicinity.
2) Transmission of relayed packets is allowed only to destination node.
   - Each packet undergoes a maximum number of two hops.
Mobile Ad Hoc Network (5/6)

- In steady-state every node has packets buffered for every other node
  - The scheduling algorithm $\pi_2$ goes as:
    1) The nodes are divided into potential receivers and transmitters based on parameter $\theta \in (0,1)$
    2) Randomly pick one of the possible $\binom{n}{n_s}$ equally likely partitions, where $n_s = n\theta$
    3) Each of the $n_s$ nodes transmits to its closest receiving node if the SINR condition is met
  - For one S-D pair, the direct route and the $n-2$ relayed routes have a throughput of $\Theta(1/n) \Rightarrow$ Total throughput per-node $\Theta(1)$

Mobile Ad Hoc Network (6/6)

- Conclusion from the results of [Gro02]:
  - By assuming
    - Infinite length buffers and information streams
    - Users that are moving and stay in same area
    - Very loose delay constraints
    - Larger number of users (“dense” network)
  - There exists $c \in \mathbb{R}^+$ such that:
    \[ \lim_{n \to \infty} \Pr \{ \lambda(n) = cR \text{ is feasible} \} = 1 \]
  - Throughput does not go to zero with $n$ !
  - Scheduling policy $\pi_2$ can be implemented also in a distributed manner
Conclusions

• Capacity of fixed and mobile wireless ad hoc networks was briefly examined.

• Per-user capacity of a fixed ad hoc network goes to zero as $1/\sqrt{n}$, regardless of the scheduling policy, routing, etc..

• With loose delay constraints, the average asymptotic throughput per-node in a mobile ad hoc network is $\Theta(1)$, that is, it is constant and strictly non-zero.